CH4 NEUTRINO MASSES

→ The masses of the fermions in the SM come from Yukawa interactions with the Higgs boson:

$$\mathcal{L}_y = y\,\bar{L}\,\phi\, e_R + h.c. = y^*\,\bar{e}_R\,\phi^\dagger L \quad \text{with } L = \begin{pmatrix}\nu_e \\ e^-\end{pmatrix}_L,\ \phi = \begin{pmatrix}\phi^+ \\ \phi^0\end{pmatrix}$$

↳ Yukawa coupling — doublets of $SU(2)$

→ The electric charge Q is given by

$Q = T_3 + Y/2$ with $T_3 = \begin{pmatrix}1/2 & 0 \\ 0 & -1/2\end{pmatrix}$ the 3rd component of the isospin and Y the hypercharge

By convention, $Y_{L_e} = -1$ and $Y_\phi = +1$

→ For the 3 generations of leptons: $i = 1,2,3$ and one writes

$$\mathcal{L}_y = y_{ij}\,\bar{L}_i\,\phi\, e_{R_j} + y^*_{ij}\,\bar{e}_{R_i}\,\phi^\dagger L_j$$

For the quarks, it's slightly ≠, but same sector of the $\mathcal{L}$.

PROP | The Yukawa coupling gives a mass to charged leptons only through the vev (vacuum expectation value) of ϕ: the Higgs mechanism.
→ Neutral leptons (neutrinos) are massless.

4.1 Standard-Model Extension

DEF | The Higgs conjugate $\tilde{\phi}$ is given by $\tilde{\phi} \equiv i\sigma^2 \phi^*$.
Its hypercharge is $Y_{\tilde{\phi}} \equiv -1$

→ explicitly, $\tilde{\phi} = i\sigma^2\phi^* = \begin{pmatrix}\phi^{0*} \\ -\phi^{+*}\end{pmatrix}$ (Recall $\phi = \begin{pmatrix}\phi^+ \\ \phi^0\end{pmatrix}$)

$\equiv \begin{pmatrix}\phi^{0*} \\ -\phi^-\end{pmatrix}$

→ Under the symmetries of the SM, $SU(3) \times SU(2) \times U(1)$,
the lepton doublet has quantum number $L_i\,(1, 2, -1)$,
→ singlet under $SU(3)$ (no interaction), doublet under $SU(2)$ (transforms under the weak interaction), with hypercharge -1
the Higgs field has $\phi\,(1, 2, 1)$

→ The combination $\bar{L}\tilde{\phi}$ has quantum numbers
$(1\otimes 1,\ 2\otimes 2,\ -1+1) = (1,\ \cancel{3}\oplus 1,\ 0) = (1, 1, 0)$

not interested

→ The combination $\overline{L}\tilde{\Phi}$ is a singlet of the SM gauge group

PROP There are 3 main types of portal interactions that connect the SM to new physics sectors.

① Higgs portal: interaction between the Higgs and new scalar particle, typically via term in the potential

$$\mathcal{L} \propto |\phi|^2 |S|^2$$

with S a dark scalar

② Vector portal: introduces a new $U(1)$ gauge boson A'_μ called dark photon that couples through kinetic mixing:

$$\mathcal{L} \propto F_{\mu\nu} F'^{\mu\nu}$$

③ Neutrino portal: introduces a new fermionic field N called sterile neutrino that couples to the SM through interactions with the ϕ and L:

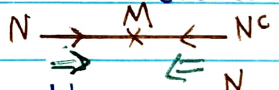
$$\mathcal{L} = y_\nu \overline{L}\tilde{\Phi} N + y_\nu^* \overline{N}\tilde{\Phi}^\dagger L$$

→ The sterile neutrino has Q# $N(1,1,0)$, and $[N] = 3/2$

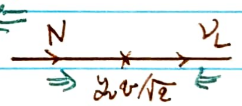
→ The vev of ϕ_0 (recall $\langle\phi\rangle = \begin{pmatrix} 0 \\ v/\sqrt{2}\end{pmatrix}$) gives mass to the lower component of the doublet, $\Leftrightarrow$ $\tilde{\Phi} = \begin{pmatrix} \phi^{0*} \\ -\phi^- \end{pmatrix}$ gives mass to upper components.

↳ Since $m_\nu < eV$, $y_\nu \frac{v}{\sqrt{2}} \overline{\nu}_L N \rightsquigarrow y_\nu$ must be tiny tiny.

→ The introduction of N gives new dof. We introduce a Majorana mass: $\mathcal{L} \ni \frac{M}{2}(\overline{N}^c N + \overline{N} N^c)$



↳ not coming from a SSB, unlike



→ Writing $M \equiv M_{maj}$ and $m \equiv y v/\sqrt{2}$, we see that the masses are not diagonales: $\mathcal{L}\big|_{SSB} = \frac{-1}{2}(\overline{\nu}_L^c \ \overline{N}) \begin{pmatrix} 0 & m \\ m & M \end{pmatrix} \begin{pmatrix} \nu_L \\ N^c \end{pmatrix} + h.c.$

Diagonalizing the matrix in the limit $M \gg m$, one finds

$\det(A - \lambda \mathbb{1}) = 0 \Leftrightarrow -\lambda(M-\lambda) - m^2 = 0 \Leftrightarrow \lambda^2 - \lambda M - m^2 = 0$

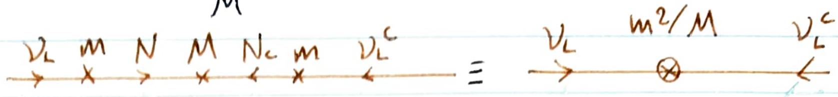
$\Leftrightarrow \lambda = \frac{M \pm \sqrt{M^2 - 4m^2}}{2} \simeq \frac{M \pm M(1 + 2m^2/M)}{2} \Rightarrow \lambda \simeq \begin{cases} M \\ -m^2/M \end{cases}$

PROP The eigenvalues and eigenvectors are mostly:

$\Psi_{light} \simeq \nu_L + \nu_L^c$ and $\Psi_{heavy} \simeq N^c + N$

with $m_{light} \simeq -m^2/M$ with $m_{heavy} \simeq M$

→ We can write $\mathcal{L} \ni -\frac{m^2}{M}\left(\bar{\nu}_L^c \nu_L + h.c.\right) + M\left(\bar{N}^c N + h.c.\right)$

Pictorially, 

DEF The see-saw mechanism is the one by which ψ_{light} acquire its mass from EWSB (like SM fermions) but is suppressed by the ratio m/M

→ The Dirac mass term violate the lepton number is $(y_\nu)_{ij}$ is non-diag.
↳ flavour eigenstate ≠ mass eigenstate
↳ only violate lepton # within each family (e, μ, τ)
The Majorana mass term violates lepton number conservation $(\Delta L = 2)$.

4.2 Experimental situations

① There is a cosmic neutrino background coming from cosmology (decoupling) ⇒ bound on its energy density

$\rho_\nu \ll H_0^2/8\pi G \Rightarrow \sum m_\nu < 45$ eV

② Other bound from the formation of large scale structure:

$0 \leq \sum m_\nu \leq 0,5$ eV

③ Constraints from lab experiments

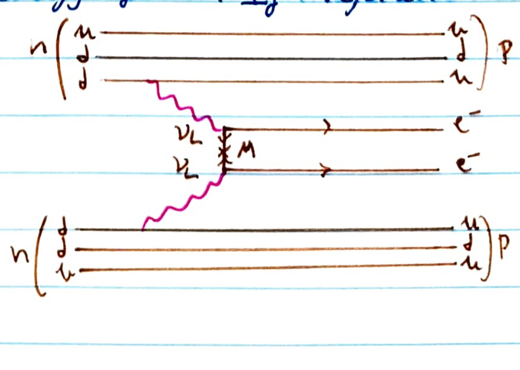
ⓐ $n \to p + e^- + \bar{\nu}_e$

If $m_{\bar{\nu}_e} > 0 \Rightarrow$ distortion in the electron spectrum $m_\nu < 1,1$ eV

ⓑ 2β-decay: $^{30}Te \to \dots + e^- + e^- + \underbrace{\bar{\nu}_e + \bar{\nu}_e}$

Look for the missing energy of ←. If Majorana mass $\Rightarrow \Delta L \neq 0$

ⓒ $n + n \to p + p + e^- + e^-$



④ Neutrino oscillation
In 1st approx., $\begin{pmatrix}\nu_\mu \\ \nu_e\end{pmatrix} = \begin{pmatrix} c\theta & s\theta \\ -s\theta & c\theta \end{pmatrix}\begin{pmatrix}\nu_2 \\ \nu_3\end{pmatrix}$

For the 3 generation, one considers the PNMS matrix.

Consider $|\psi\rangle = |\nu_\mu\rangle = \cos\theta\,|\nu_2\rangle + \sin\theta\,|\nu_3\rangle$.

Now, the time evolution is $|\nu_2(t)\rangle = e^{-iE_2 t}|\nu_2\rangle$; $|\nu_3(t)\rangle = e^{-iE_3 t}|\nu_3\rangle$

so that $|\psi(t)\rangle = \cos\theta\, e^{-iE_2 t}|\nu_2\rangle + \sin\theta\, e^{-iE_3 t}|\nu_3\rangle$

$\hookrightarrow \langle \nu_e|\nu_\mu(L)\rangle = -s\theta\, c\theta\, e^{-iE_1 L} + s\theta\, c\theta\, e^{-iE_2 L}$

$$= \sin(2\theta)/2 \cdot \left(e^{-iE_1 L} - e^{-iE_2 L}\right)$$

$$= \sin(2\theta)/2 \cdot e^{-iE.L}\left(e^{-m_1^2/2E.L} - e^{-m_2^2/2E.L}\right)$$

$\hookrightarrow P(\nu_\mu \to \nu_e) = |\langle \nu_e|\nu_\mu(L)\rangle|^2 = \sin^2(2\theta)\sin^2((E_2-E_1)L/2)$

$$= \sin^2(2\theta)\sin^2\left((m_1^2 - m_2^2)L/4E\right)$$

$$= \sin^2(2\theta)\sin^2\left(1{,}27 \cdot \frac{\Delta m^2 . L}{E}\ \frac{[eV^2][km]}{[GeV]}\right)$$

→ One can have normal ordering $m_1 < m_2 < m_3$
or inverse ordering $m_3 < m_1 < m_2$

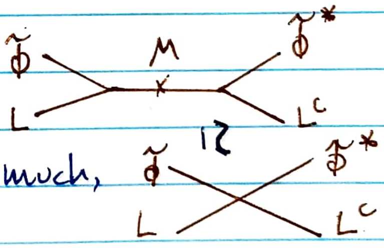
$\hookrightarrow \Delta m_{12}^2 \sim 10^{-3}\,eV^2$ or $\sim 10^{-5}\,eV^2$

4.3 Effective field theory

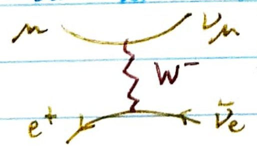
We want an effective interaction which gives masses to neutrinos once the symmetry is broken, using the building blocks of $\mathcal{L}_{SM}$: $\phi, L, Q\ldots$

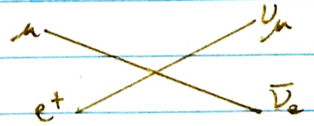
→ ex: $\bar{L}\tilde{\phi}$ is a singlet of the SM gauge group $SU(3)\times SU(2)\times U(1)$: it is gauge invariant.

→ Thanks to $\nu_L \overset{m}{\times}\overset{M}{\times}\overset{m}{\times} \nu_L^c$, we can write
Assuming that the heavy dof don't propagate much,



→ Same as Fermi theory for weak interaction



$\equiv \frac{1}{q^2 - M_W^2} \sim \frac{-1}{m_W^2} \equiv$


→ The effective Lagrangian $\mathcal{L}_{eff}$ reads $\mathcal{L}_{eff} = \frac{y^2}{M}\underbrace{\bar{L}\tilde{\phi}L^c\tilde{\phi}^*}_{\text{Weinberg operator}}$

→ $[\phi] = 1$, $[L] = 3/2$ so that $[\bar{L}\tilde{\phi}L^c\tilde{\phi}^*] = 5$, combined with $1/M$ in front
$[\mathcal{L}_{eff}] = 4$

PROP Operators in $\mathcal{L}$ with dimension ≤ 4 are renormalizable
> 4 are non-renormalizable

→ When using the Yukawa interaction and the Majorana mass for neutrinos, we have a renormalizable theory.
Once one integrates out the dof corresponding to the Majorana mass, the interaction is no more renormalizable.

→ Consider a theory with a heavy scalar field interacting with light fermions

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}M^2\phi^2 + \bar\psi(i\not\partial - m)\psi + y\bar\psi\phi\psi$$

↳ we want to write an effective theory involving only ψ and $\partial_\mu\psi$: we integrate out the heavy field ϕ. Since $M \gg m$, ϕ doesn't propagate much at low energy. We want to write $\phi = \phi(\psi, \partial_\mu\psi)$

1) EOM: $\begin{cases} \delta_\phi S = 0 \Leftrightarrow (\partial^2 + M^2)\phi = y\bar\psi\psi \\ \delta_{\bar\psi} S = 0 \Leftrightarrow (i\not\partial - m)\psi = -y\phi\psi \end{cases}$

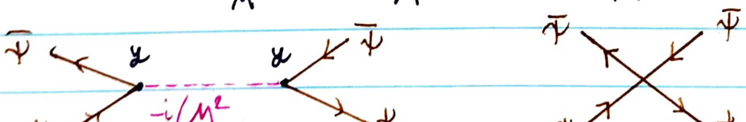
$$\Rightarrow \mathcal{F}(\phi) = \frac{-y\bar\psi\psi}{p^2 - M^2} \Rightarrow \phi = \frac{y\bar\psi\psi}{\partial^2 + M^2}$$

2) Expand: consider $p^2 \ll M^2$ so that

$$\phi = \frac{y\bar\psi\psi}{M^2} - \frac{y}{M^4}\partial^2\bar\psi\psi + \frac{1}{M^2}\mathcal{O}\left((\partial^2/M^2)^2\right)$$

3) Substitute: $\phi \mapsto y\bar\psi\psi/M^2 - y\partial^2\bar\psi\psi/M^4$. One gets

$$\mathcal{L}_{eff}\big|_{mass} = -\frac{1}{2}M^2\frac{y^2}{M^4}(\bar\psi\psi)^2 + \frac{y^2}{M^2}(\bar\psi\psi)^2 = \frac{1}{2}\frac{y^2}{M^2}\bar\psi\psi\bar\psi\psi + \mathcal{O}(M^{-4})$$



→ We now have $\mathcal{L}_{eff} = \bar\psi(i\not\partial - m)\psi - \frac{1}{2}y^2/M^2\,\bar\psi\psi\bar\psi\psi$ that contains a effective 6-dim. operator.

Higher order contribution $1/2\, y^2/M^2\,\partial_\mu(\bar\psi\psi)\partial^\mu(\bar\psi\psi) + y^2/M^4\,\bar\psi\psi\partial^2\bar\psi\psi$
$= \frac{1}{2}y^2/M^4\,\bar\psi\psi\partial^2\bar\psi\psi$ + Boundary

PROP Any QFT is an effective field theory (EFT) valid in a limited range of energy.

4.4 Path integral formalism

→ In case one forgot their course of QFT 2 and ad QFT, let's review one more time the PI formalism.

→ Consider the propagation from q_i to q_f in a time T

→ The amplitude is given by:

$$\langle q_f | e^{-iHt} | q_i \rangle = \langle q_f | e^{-iH\delta t} \cdots e^{-iH\delta t} | q_i \rangle$$
$$= \int dq_1 \cdots dq_N \, \langle q_f | e^{-iH\delta t} | q_N \rangle \langle q_N | \cdots | q_1 \rangle \langle q_1 | e^{-iH\delta t} | q_i \rangle$$

→ Focus on one factor:

$$\langle q_1 | e^{-iH\delta t} | q_i \rangle = \int dp_1 \langle q_1 | p_1 \rangle \langle p_1 | e^{-iH\delta t} | q_i \rangle$$
$$= \int dp_1 \langle q_1 | p_1 \rangle e^{-ip_1^2/2m \cdot \delta t} \, e^{-ip_1 q_i} \, e^{-iV(q_i)\delta t}$$
$$= \int dp_1 \exp\{-i(\tfrac{p_1^2}{2m} + V(q_i))\delta t\} \exp\{i p_1 \dot{q}_1 \delta t\}$$
$$= \int dp_1 \exp\{-i(H(q_1, q_i) - p_1 \dot{q}_1)\delta t\}$$

→ The total amplitude can be rewritten as

$$\langle q_f | e^{-iHt} | q_i \rangle = N \int dq_1 \cdots dq_N \left(e^{i(m\dot{q}_1^2/2 - V(q_1))\delta T} \cdots e^{i(m\dot{q}_N^2/2 - V(q_N))\delta T} \right)$$
$$= N \int \prod dq(t) \, e^{i\int L dt}$$
$$= N \int \mathcal{D}q(t) \, e^{iS} \quad \text{with} \begin{cases} q(t_i) \equiv q_i \\ q(t_f) \equiv q_f \end{cases}$$

→ The dominating path is the classical path.

DEF The generating function Z is defined as

$$Z \equiv \langle 0_+ | 0_- \rangle_J = N \int \mathcal{D}\phi \, \exp\{iS[\phi, \partial_\mu \phi] + i\int d^4x \, \phi J\}$$
$$= e^{iW}$$

where J is the source and W the generating functional for connected Green's functions.

→ The source J is a convenient tool to compute amplitudes of operators.

DEF The correlation function $\langle \cdots \rangle$ is defined as

$$\langle \phi_1 \cdots \phi_n \rangle \equiv \frac{\int \mathcal{D}\phi \, \phi(x_1) \cdots \phi(x_n) \, e^{iS}}{\int \mathcal{D}\phi \, e^{iS}}$$

PROP $$\langle \phi_1 \cdots \phi_n \rangle = \frac{(-i)^n}{Z} \frac{\delta}{\delta J(x_1)} \cdots \frac{\delta}{\delta J(x_n)} Z \Big|_{J=0} = \frac{N}{Z} \int \mathcal{D}\phi \, \phi(x_1) \cdots \phi(x_n) \, e^{iS + i\int J\phi} \Big|_{J=0}$$

⊙ Effective operator:

→ PI ≡ integration over all configuration of the field.

→ In EFT, we split the integration in a sum over smooth configurations ($k^2 \ll \Lambda^2$) denoted ϕ_L and sharp configurations ($k^2 \gg \Lambda^2$) denoted ϕ_H. Then, we integrate out ϕ_H

→ We assume $J = J_L$: the source can only excite low energy modes.

$$Z[J_L] = N \int D\phi_L \, D\phi_H \, \exp\{ iS[\phi_L, \phi_H] + i \int d^4x \, \phi_L J_L \}$$

$$= N' \int D\phi_L \exp\{ i \tilde{S}[\phi_L] + i \int d^4x \, \phi_L J_L \}$$

where we wrote $\tilde{S}[\phi_L] \equiv S_L[\phi_L] + \Delta S_{eff}[\phi_L]$

↳ the action $\tilde{S}$ contains effective operators // $(\partial^2 + m^2)^{-1} \simeq m^{-2} - \partial^2/m^2$

PROP We can rewrite the effective action $\tilde{S} = S_{eff}$ as

$$S_{eff} = \sum_i \frac{1}{\Lambda^{\gamma_i}} c_i \, \mathcal{O}_i$$

where: $\mathcal{O}_i$ are effective operators

Λ is the cutoff scale

γ^i is a suppression exponent

↳ $[\mathcal{O}_i] = D + \gamma^i$ so that $\gamma_i = \delta_i - D$ where δ_i is the bare mass dimension of the operator $\mathcal{O}_i$.

→ For non-renormalizable operators ($\Leftrightarrow \gamma^i > 0$), the amplitudes goes as $\mathcal{M} \sim (E/\Lambda)^{\gamma_i}$, so is suppressed for $E \ll \Lambda$

$\mathcal{O}_i$	δ_i	γ_i	$\mathcal{M}_i$
$\partial_\mu \phi \partial^\mu \phi$	D	0	1
ϕ	$(D-2)/2$	/	/
ϕ^4	$2D-4$	$D-4$	$(E/\Lambda)^{D-4}$
ϕ^2	$D-2$	-2	$(\Lambda/E)^2$
V_0	0	$-D$	$(\Lambda/E)^D$
ϕ^6	$3D-6$	$2D-6$	$(E/\Lambda)^{2D-6}$

Usually, marginal operators are associated with interactions.

DEF We call marginal operator the one such that $\gamma_i = 0$, relevant operator if $\gamma_i < 0$ and irrelevant operator if $\gamma_i > 0$

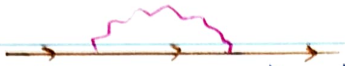
→ Consider a scalar field in $D=4$. The effective action reads

$$S_{eff} = \int d^4x \left\{ \frac{1}{2}\partial_\mu \phi_L \partial^\mu \phi_L - \frac{1}{2}\underbrace{c_2 \Lambda^2}_{m^2} \phi_L^2 - \frac{\lambda}{4!}\phi_L^4 + \frac{c_6}{\Lambda^2}\phi_L^6 + c_0 \Lambda^4 + \dots \right\}$$

↳ the mass of the theory is $m \sim \Lambda$

→ In QED, the correction to the mass is $\propto$ to the mass itself:

$$\delta m_e \propto m_e (\log \Lambda)$$



↳ When $m_e = 0$, the theory has a chiral symmetry

↳ The gauge symmetry prevents the field to get a mass: $\delta m_\gamma = 0$

→ We expect $m_\phi \sim \Lambda$, but we observe really low.

↳ naturalness problem

⊙ Naturalness problem of the SM:

→ The higgs field has a quadratic sensitivity to quantum correction:

$$\delta m_\phi \sim \Lambda^2$$

For $\Lambda \sim m_{PL} \sim 10^{19}$ GeV, we expect δm_H^2 to become extremely large

→ Unlike fermions, the Higgs is not protected by a chiral symmetry, nor has a gauge symmetry like vectors.

→ Solutions exists:

1) Composite Higgs Models
   ↳ Λ = scale at which we see components
2) Pseudo-Goldstone Boson:
   ↳ initially massless, then SSB
3) Super-Symmetry:
   ↳ quantum corrections from superpartners cancel the previous one.

⊙ Running of couplings:

→ Iterating the splitting of the action on $\Lambda - d\Lambda < k \leq \Lambda$ gives rise to an update of the S_{eff}: $S_{eff}(c_i(\Lambda)) \mapsto S_{eff}(c_i(\Lambda - d\Lambda)) \equiv S_{eff}(c_i(\Lambda'))$

DEF The energy dependence of the coupling c_i is governed by the β-function:

$$\beta(c_i) \equiv \Lambda \frac{dc_i}{d\Lambda}$$